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Abstract

This paper considers VAR/VECM models for variables exhibiting cointegration and common features in the transitory components. While the presence of cointegration reduces the rank of the long-run multiplier matrix, other types of common features lead to rank reduction in the short-run dynamics. These common transitory components arise when linear combination of the first differenced variables in a cointegrated VAR are white noise. This paper offers a reinterpretation of the traditional approach to testing for common feature dynamics, namely checking for a singular covariance matrix for the transitory components. Instead, the matrix of short-run coefficients becomes the focus of the testing procedure thus allowing a wide range of tests for reduced rank in parameter matrices to be potentially relevant tests of common transitory components. The performance of the different methods is illustrated in a Monte Carlo analysis which is then used to reexamine an existing empirical study. Finally, this approach is applied to analyze whether one would observe common dynamics in standard DSGE models.

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Transitory components, common features, reduced rank, cointegration.

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1 Introduction

Cointegration between series implies that there are common factors among their permanent components and much literature has been devoted to the twin issues of determining how many common factors there are and extracting them from data. Vahid and Engle (1993) extended this idea by asking whether there were common factors in the transitory components of series. The predominant approaching to testing for common transitory components so far is the Likelihood Ratio test (Vahid and Engle, 1993; Hecq, 2004; Hecq et al., 2000a, 2006). Vahid and Engle (1993) also describe a Lagrange Multiplier (LM) test but this is used less often in the literature merely because the restricted VECM model is more difficult to estimate than its unrestricted counterpart. In general, imposing common feature restrictions, when they are appropriate, will result in an increase in estimation efficiency (Lütkepohl, 1991) and in the accuracy of forecasts (Vahid and Issler, 2002). Testing for the number of common features therefore remains a relevant question of interest.

This paper makes three contributions to the existing literature on testing for common feature dynamics. First, it offers a reinterpretation of the approach to determining whether or not there are common factors in the transitory components by checking the rank of the matrix containing the coefficients that summarize the short-run dynamics of the system. This opens the way for tests on the rank of a matrix of parameters to be used to determine the the validity of common feature restrictions. A good deal of work exists on how one tests the rank of a non-square matrix of parameters, with notable contributions being by Cragg and Donald (1993, 1996, 1997), Robin and Smith (2000) and Kleibergen and Paap (2006). Second, it introduces a Wald test of the hypothesis that there are common dynamics and shows that this test is asymptotically the same as the popular LR test. This test is easy to implement as it requires estimation of the unrestricted model only. It is also demonstrated that when applied in the setting of a VAR/VECM the Wald test is identical to the the Cragg and Donald (1993, 1997) minimum discrepancy test, the Robin and Smith (2000) characterstic root test and the Kleibergen and Paap (2006) singular value decomposition test, thus providing a link between the traditional approach to the common features problem and the literature on testing the rank of a matrix. Third, it demonstrates how this reinterpretation of the common transitory components literature also leads to a way of checking if calibrated Dynamic Stochastic General Equilibrium (DSGE) models have common dynamics.

The rest of this paper is structured as follows. Section 2 sets out the re-interpretation of tests for common feature dynamics and Section 3 shows how this involves testing the rank of the matrix of short-run dynamics coefficients. In Section 4 the Wald test is described along with other tests for reduced rank. The relationships between the tests of the two different approaches are established. Section 5 then conducts a simulation study on the relative efficacy of the LR and Wald tests, finding that the latter seems to have higher (size-corrected) power in small samples, although both tests show significant size distortion. The simulation results are then used to guide an empirical investigation into common dynamics among Latin American GDP series. Section 6 turns to the issue of how to apply these tests to models that feature a good deal in quantitative macroeconomic work today, namely DSGE models. In these models non-stationarity is often handled by working with a transformed model in which the integrated variables have been transformed to achieve stationarity. The mapping between this transformed system and a Vector Error Correction Model (VECM) is derived and used to consider whether there are common dynamics in the DSGE model. Section 7 is a brief conclusion.

2 Common Factors in Transitory Components

Let y_t be an $n \times 1$ vector at time t that can be represented as a vector autoregression of order p

$$y_t = \Pi_1 y_{t-1} + \cdots + \Pi_p y_{t-p} + \varepsilon_t, \tag{1}$$

where ε_t is a white noise process. Equation (1) can be re-parameterized as the following VAR of order $p-1$

$$\Delta y_t = \Pi y_{t-1} + A_1 \Delta y_{t-1} + \cdots + A_{p-1} \Delta y_{t-p1} + \varepsilon_t. \tag{2}$$

where

$$\Pi = I_n - \sum_{i=1}^{p} \Pi_i \,, \qquad A_j = -\sum_{i=j+1}^{p} \Pi_i \,.$$

If the rank of the matrix Π is $r < n$ then it is usually expressed as $\Pi = \alpha\beta'$, where α and β are both $n \times r$ matrices of rank r and the VAR becomes a VECM process. The β' matrix contains the cointegrating vectors along its rows, while the columns of α contain the adjustment coefficients of each variable in y_t to a particular cointegrating vector. This paper will assume that the cointegrating rank, r, is able to be determined by ignoring any restrictions on the short-run dynamics of the model, and that the super-consistent estimates of β may for all practical purposes be treated as fixed.

Common transitory components were then defined by Engle and Kozicki (1993) and Vahid and Engle (1993) as occurring if there existed an $n \times s$ matrix τ, called the cofeature matrix, such that

$$\tau' \Delta y_t = e_t \, , \tag{3}$$

where e_t was white noise. In other words, there existed s linear combinations of Δy_t that are white noise processes. Combining this definition with that of Δy_t in (2) under co-integration, indicates that the existence of common components means that the cofeature matrix τ must satisfy two conditions:

$$\begin{aligned} \tau' A_i &= 0 \, , \qquad i = 1, \cdots, p-1 \\ \tau' \alpha \beta' &= 0 \, . \end{aligned}$$

Essentially these conditions require that τ lies in the intersection of the null spaces of the matrices describing the short-run dynamics of the system. Vahid and Engle (1993) point out that τ is only identified up an invertible transform and therefore suggest that τ' be expressed in reduced row echelon form to ensure that there are enough exclusion-normalization restrictions to identify it uniquely. The general structure of τ when τ' is expressed in reduced row echelon will be the $n \times s$ matrix

$$\tau = \begin{bmatrix} I_s \\ \tau^*_{(n-s)\times s} \end{bmatrix} . \tag{4}$$

The VECM which holds in the presence of co-integration will be

$$\Delta y_t = \Phi' \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \beta' y_t \end{bmatrix} + \varepsilon_t \, , \tag{5}$$

where

$$\Phi = \begin{bmatrix} A_1' \\ \vdots \\ A_{p-1}' \\ \alpha' \end{bmatrix} \tag{6}$$

is a $k \times n$ matrix of parameters with $k = n(p-1)+r$. From this it is clear that an alternative approach to checking for common transitory components is to focus on the rank of the matrix of the parameters Φ. In this representation, the condition in equation (3) will be satisfied if and only if

$$\tau'\Phi' = 0\,.$$

This condition requires that the $k \times n$ matrix of parameters Φ has a nontrivial null-space. Since $k > n$ the rank of Φ is at most n and, by the rank-nullity theorem,

$$\text{rank}(\Phi') + \text{nullity}(\Phi') = n\,,$$

the existence of the $n \times s$ cofeature matrix τ requires that Φ must have rank $q = n - s$. The existence of common transitory components therefore manifests itself in the reduced rank of Φ. This representation has the advantage that it points to the need to examine the rank of the matrix Φ and a substantial literature exists on testing the rank of such a matrix.

Some restrictions may need to be imposed to identify the s common transitory components. The restricted system comprises pseudo-structural equations for the first s elements of the vector Δy_t which correspond to the set of identified common components. The last $n - s$ equations in the system are simply the reduced-form equations for the remaining elements of Δy_t. The model can be expressed as

$$B'\Delta y_t = \begin{bmatrix} & 0_{s\times k} & & \\ A_1^* & \cdots & A_{p-1}^* & \alpha^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \beta' y_t \end{bmatrix} + B'\varepsilon_t\,, \tag{7}$$

where

$$B = \begin{bmatrix} I_s & 0_{s\times(n-s)} \\ \tau^*_{(n-s)\times s} & I_{n-s} \end{bmatrix}$$

and the parameter matrices A_i^* and α^* are the remaining $n - s$ rows of their unrestricted counterparts in equation (5).

3 The Beveridge-Nelson Decomposition

Quite a large literature has emerged that has tested for whether there are common factors in the transitory components of a variety of contexts including: property markets (Wang, 2003; Liow, 2007); stock markets, Hecq et al. (2000b); and Asian and Latin American economic activity (Sato et al., 2007, Hecq, 2004). Generally, this literature has been referred to as testing for common cycles, with the assumption that the transitory component measures the cycle. This is incorrect unless one is referring to the growth cycle so the more neutral description of testing for common transitory components will be used in this paper.

The Beveridge-Nelson definition of the permanent component of y_t is $y_t^P = E_t y_\infty$. Since

$$\begin{aligned} y_t^P &= E_t y_\infty = E_t(y_t + \sum_{j=1}^{\infty} \Delta y_{t+j}) \\ &= y_t + E_t(\sum_{j=1}^{\infty} \Delta y_{t+j}), \end{aligned}$$

the transitory component of y_t will be

$$y_t^T = y_t - y_t^P = -E_t(\sum_{j=1}^{\infty} \Delta y_{t+j}). \tag{8}$$

Now it follows from the definition of y_t^P that $\Delta y_t^P = \eta_t$, where η_t is the new information in predicting y_∞ and $E_{t-1}(\eta_t) = 0$ i.e. η_t is a martingale difference sequence (or white noise process). Expressing y_t in first-difference terms we then have

$$\Delta y_t = \Delta y_t^P + \Delta y_t^T .$$

Clearly, if there exists a matrix τ such that $\tau' \Delta y_t^T = 0$ then

$$\tau' \Delta y_t = \tau' \eta_t = e_t$$

i.e. there is a linear combination of the Δy_t that is white noise, which is exactly the condition tested by Vahid and Engle (1993).

Rather than check for whether there is a combination of Δy_t that is white noise we could ask if there exists a τ such that $\tau' \Delta y_t^T = 0$. From equation (8), this would mean that

$$\tau' \Delta y_t^T = -\tau' E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = 0 \tag{9}$$

Consider the VECM written in companion form

$$\begin{bmatrix} \Delta y_t \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+2} \\ z_t \end{bmatrix} = \Phi^* \begin{bmatrix} \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0_{n\times 1} \\ \vdots \\ 0_{n\times 1} \\ \beta'\varepsilon_t \end{bmatrix}, \tag{10}$$

where $z_t = \beta' y_t$ is the error-correction (EC) part of the model and the companion form matrix Φ^* is given by

$$\Phi^* = \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-2} & A_{p-1} & \alpha \\ I_n & 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times r} \\ 0_{n\times n} & I_n & & 0_{n\times n} & 0_{n\times n} & 0_{n\times r} \\ \vdots & & \ddots & & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & & I_n & 0_{n\times n} & 0_{n\times r} \\ \beta' A_1 & \beta' A_2 & \cdots & \beta' A_{p-2} & \beta' A_{p-1} & (I_r + \beta'\alpha) \end{bmatrix}. \tag{11}$$

Letting

$$W_t = \begin{bmatrix} \Delta y_t \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+2} \\ z_t \end{bmatrix},$$

(10) can be written as

$$W_t = \Phi^* W_{t-1} + \varepsilon_t^*,$$

from which it follows that $E_t W_{t+j} = \Phi^{*j} W_t$. Therefore

$$\Delta y_t^T = E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = E_t \sum_{j=1}^{\infty} S\Phi^{*j} W_t \tag{12}$$

$$= S\Phi^* (I - \Phi^*)^{-1} W_t, \tag{13}$$

where S is a selection matrix such that $\Delta y_t = SW_t$ and it is assumed that the eigenvalues of Φ^* are such that infinite sum $\sum_{j=0}^{\infty} \Phi^{*j}$ converges to $(I - \Phi^*)^{-1}$.

From (**??**), $\tau' \Delta y_t^T = 0$ requires that $\tau' S\Phi^* = 0$ i.e. $\tau'\Phi = 0$. This is exactly the condition established in the previous section. Hence any singularity in the transitory components y_t^T manifests itself in a rank deficiency of Φ, so this suggests an alternative way of detecting common factors in the transitory components is to examine the rank of Φ .

4 Testing for Common Transitory Components

The following tests all investigate the null hypothesis of the presence of s common transitory components where $s = 1, \ldots, n - r$. In the first set one is essentially testing whether the short-run dynamics can be removed from the system and asks the question of whether it is possible to find a combination of the Δy_t that is white noise. In the second set one directly tests whether Φ has rank $n - s$. As can be seen from the pseudo-structural form of the system in expression (7), under the null of s common transitory components, sk restrictions are placed upon the parameters in Φ but $s(n - s)$ new parameters τ^* must be introduced in order to uniquely identify the cofeature vector τ. Therefore for known r and β, all test statistics have an asymptotic χ^2 distribution with $sk - s(n - s)$ degrees of freedom.

4.1 Tests using the Cofeature Vector

4.1.1 The Vahid-Engle LM Test

Define the $T \times k$ (where $k = n(p-1) + r$) matrix $W_2 = [\Delta y'_{-1}, \ldots, \Delta y'_{-p+1}, Y'_{-1}\beta]$ which contains the stacked observations of all the relevant lagged values of the system. Vahid and Engle (1993) suggested estimating τ by means of the LIML estimator, where W_2 were used as instruments for Δy_t. The test statistic is then

$$\chi_{LM} = TR^2, \tag{14}$$

where the R^2 is obtained from the auxiliary regression of the residuals, $\widehat{\tau}'\Delta y_t$, on the instruments W_2, with $\widehat{\tau}$ normalized to be in reduced row-echelon form. This test has not been used extensively in the applied literature, possibly due to the need to estimate τ first, followed thereafter by an auxiliary regression.

4.1.2 Likelihood Ratio Test

The test used most commonly in applied work is the Likelihood Ratio test of Vahid and Engle (1993), based on the smallest canonical correlations between Δy_t and the relevant past of the process. This test is the canonical correlations test of Anderson (1951) specialized to a VECM.

Let $\Delta y_t' = [\Delta y_{1t} \cdots \Delta y_{nt}]$ be a $1 \times n$ vector of variables and $\varepsilon_t' = [\varepsilon_{1t} \cdots \varepsilon_{nt}]$ be a $1 \times n$ vector of disturbances. Now define the matrix W_1 as a $T \times n$ matrix formed by stacking the observations on Δy_t.[1] Let $0 \leq \widehat{\nu}_1 \leq \widehat{\nu}_2 \leq \cdots \leq \widehat{\nu}_n \leq 1$ be the ordered eigenvalues of the $n \times n$ symmetric matrix[2]

$$\Psi = (W_1'W_1)^{-1}W_1'W_2(W_2'W_2)^{-1}W_2'W_1 \,. \tag{15}$$

The test statistic is then defined as

$$\xi_{LR} = -T \sum_{i=1}^{s} \log(1 - \widehat{\nu}_i) \,. \tag{16}$$

4.1.3 Wald Test

To date it has not been recognized in the literature that a Wald test of the common feature restrictions is also available. For this purpose, it is convenient to write the unrestricted system as

$$W_1 = W_2\Phi + \varepsilon \,, \tag{17}$$

and ε is now a $T \times n$ matrix of the stacked disturbances ε_t'. The maximum likelihood estimator $\widehat{\Phi}$ of Φ is easily obtained and the distribution of $\widehat{\Phi}$ is given by

$$\sqrt{T}(\mathrm{vec}\,\widehat{\Phi} - \mathrm{vec}\,\Phi) \xrightarrow{d} \mathrm{N}(0, \Sigma) \tag{18}$$

where $\widehat{\Sigma}$ is a $\sqrt{T}$-consistent estimator of $\Sigma = \Omega \otimes Q^{-1}$ (Hamilton, 1994) with Ω representing the covariance matrix of the disturbance terms ε and Q^{-1} representing the covariance matrix of W_2.[3]

The common feature restrictions in equation (3) are just a set of linear restrictions once the cofeature matrix τ used in the construction of the matrix B is prescribed. An estimate $\widehat{\tau}$ of

[1] Note that if the VECM (2) has an intercept term, this will need to be removed from the series when constructing W_1 and W_2.

[2] This form for Ψ was suggested by Anderson and Vahid (1998). Hecq *et al.* (2006) use the eigenvalues of $(W_1'W_1)^{-1/2}W_1'W_2(W_2'W_2)^{-1}W_2'W_1(W_1'W_1)^{-1/2}$ as the canonical correlations for their test. The eigenvalues of both matrices are the same.

[3] For a VECM with an intercept term in expression (2), $\widehat{\Sigma} = \widehat{\Omega} \otimes \widehat{Q}_0^{-1}$ where $\widehat{\Omega}$ is an estimator of $\mathrm{E}[\varepsilon_t\varepsilon_t']$ and $\widehat{Q}$ an estimator of $\mathrm{E}[\,X_tX_t']$, with $X_t = (1, \Delta Y_{t-1}', \ldots, \Delta Y_{t-p+1}', Y_{t-1}'\beta)'$. Then $\widehat{Q}_0^{-1}$ is constructed from $\widehat{Q}^{-1}$ by removing the first row and column of $\widehat{Q}^{-1}$.

τ is obtained as a by-product of the LR test. As pointed out by Vahid and Engle (1993), the s cofeature vectors are the eigenvectors corresponding to the s smallest eigenvalues of the matrix Ψ given in equation (15). Once $\widehat{\tau}'$ has been expressed in reduced row echelon form, the matrix B may be constructed.

By comparing the unrestricted and restricted models in equations (5) and (7), it is clear that the restrictions hold if $\Phi\tau$ is a zero matrix. The reduced-rank restrictions can therefore be written in the form

$$R\,\mathrm{vec}\,\Phi = 0_{sk\times 1} \qquad R = \tau' \otimes I_k\,.$$

Using an estimate $\widehat{\tau}$ for τ obtained from the eigendecomposition of Ψ in expression (15), the Wald test statistic of these reduced-rank restrictions is

$$\begin{aligned}\xi_W &= \big[R\,\mathrm{vec}\,\widehat{\Phi}\,\big]'\big[R\,\mathrm{Var}(\mathrm{vec}\,\widehat{\Phi}\,)R'\big]^{-1}\big[R\,\mathrm{vec}\,\widehat{\Phi}\,\big] &(19)\\ &= \big[R\,\mathrm{vec}\,\widehat{\Phi}\,\big]'\big[\frac{1}{T}R\,\widehat{\Sigma}\,R'\big]^{-1}\big[R\,\mathrm{vec}\,\widehat{\Phi}\,\big]\,. &(20)\end{aligned}$$

In the Appendix it is shown that

$$\xi_W = T\sum_{i=1}^{s}\widehat{\lambda}_i = T\sum_{i=1}^{s}\frac{\hat{\nu}_i}{1-\hat{\nu}_i}\,,$$

a result which demonstrates that the Wald test is asymptotically equivalent to the the LR test.

4.2 Matrix Reduced Rank Tests

There is now a substantial literature on testing the rank of a rectangular matrix of parameters. Examples are the Singular Value Decomposition (SVD) test of Kleibergen and Paap (2006), the Minimum Discrepancy test of Cragg and Donald (1993, 1997) and the Characteristic Root test of Robin and Smith (2000). The main requirement for implementing these tests is that a consistent estimate of the matrix, in this case Φ, and its covariance matrix, Σ, are available.[4] In this situation the distribution results presented in equation (18) are relied on.

[4]Cragg and Donald (1996) propose a procedure for testing the rank of Φ based on a transformation of Φ using Gaussian elimination. This test was found to perform very poorly in the context of VAR/VECM models and the reasons for this remain to be investigated.

4.2.1 Singular Value Decomposition (SVD) Test

Kleibergen and Paap (2006) followed Ratsimalahelo (2002) in proposing a test for the rank of Φ based on its SVD. Whilst the test of Ratsimalahelo (2002) is based on the SVD of $\widehat{\Phi}$, Kleibergen and Paap (2006) advocated using a scaled version $\widehat{\Theta} = M\widehat{\Phi}N$, where M and N are $k \times k$ and $n \times n$ non-singular matrices chosen such that the estimated asymptotic variance of $\operatorname{vec}\widehat{\Theta}$ approximates the identity matrix. They suggested that this transformation yields superior power and numerical accuracy. Preliminary evidence suggested that this was indeed the case, so the Kleibergen and Paap (2006) variant of the test will be employed in this paper.

Let the SVD of the matrix $\widehat{\Theta}$ be $\widehat{\Theta} = U\Lambda V'$ where Λ is a diagonal matrix containing the singular values of $\widehat{\Theta}$ in descending order along its leading diagonal and zeros elsewhere. The matrices U, S and V are then all partitioned conformably so that

$$U = [U_1 \quad U_2] \qquad \Lambda = \operatorname{diag}\{\Lambda_1, \Lambda_2\} \qquad V = [V_1 \quad V_2]$$

where U_2 has s columns, Λ_2 is $s \times s$ containing the s smallest singular values of Φ, and V_2' has s rows. If $\operatorname{rank}\Phi = n-s$, then Λ_2 should be a zero matrix. The test statistic for whether the the singular values of Λ_2 are statistically different from zero is

$$\xi_{SVD} = T\,(\operatorname{vec}\Lambda_2)' \left[(V_2' \otimes U_2')\,\text{Est. Asy. Var}(\operatorname{vec}\widehat{\Theta})\,(V_2 \otimes U_2)\right]^{-1} \operatorname{vec}\Lambda_2\,. \tag{21}$$

When estimating the parameters of a VECM, the Kronecker structure of Σ means that appropriate scaling matrices M and N are readily available. Choosing $M = Q^{1/2}$ and $N = \Omega^{-1/2}$ yields the desired result that the estimated asymptotic variance of $\operatorname{vec}\widehat{\Theta}$ is the identity matrix.

4.2.2 Minimum Discrepancy Test

Cragg and Donald (1993, 1997) proposed a minimum discrepancy test based on the distance between the estimated value of the parameter matrix $\widehat{\Phi}$, which is almost surely full rank, and the matrix of rank $n-s$ that is “nearest” to $\widehat{\Phi}$.

Define $C = \{\Upsilon \in (\mathbb{R}^k)^n : \operatorname{rank}\Upsilon = n-s\}$. If $\operatorname{rank}\Phi = n-s$ then under the null hypothesis of s common transitory components there should exist a matrix in $\Upsilon_0 \in C$ that is approximately equal to $\widehat{\Phi}$ to within sampling error. Using this rationale, Cragg and Donald (1993, 1997) defined a test statistic that is the minimum possible discrepancy between $\widehat{\Phi}$ and its reduced-rank counterpart, namely

$$\xi_{MD} = T \min_{\Upsilon \in C} \operatorname{vec}(\widehat{\Phi} - \Upsilon)' \widehat{\Sigma}^{-1} \operatorname{vec}(\widehat{\Phi} - \Upsilon)\,. \tag{22}$$

4.2.3 Characteristic Root Test

Robin and Smith (2000) proposed a procedure to indirectly test the rank of Φ by examining $A\Phi' B\Phi$, where A and B are respectively $n \times n$ and $k \times k$ non-singular matrices, ensuring that $\operatorname{rank} \Phi = \operatorname{rank} A\Phi' B\Phi$. If $\operatorname{rank} \Phi = n - s$, then the s smallest eigenvalues of $\widehat{A}\widehat{\Phi}'\widehat{B}\widehat{\Phi}$ should be zero to within sampling error.

In general, the choice of A and B is arbitrary. However, when the asymptotic covariance matrix of $\operatorname{vec}\widehat{\Phi}$ takes the form $\Sigma = \widehat{\Omega} \otimes \widehat{Q}^{-1}$, as is the case when estimating the parameters of a VECM, then choosing $\widehat{A} = \widehat{\Omega}^{-1}$ and $\widehat{B} = \widehat{Q}$ greatly simplifies the expression and distribution of the test statistic.

Let $0 \leq \widehat{\lambda}_1 \leq \widehat{\lambda}_2 \leq \ldots \leq \widehat{\lambda}_n$ denote the ordered eigenvalues of $\widehat{A}\widehat{\Phi}'\widehat{B}\widehat{\Phi}$. Let $h(\lambda)$ be any function with continuous first derivatives satisfying $h(\lambda) \geq 0$ for $0 \leq \lambda < \infty$, $h(0) = 0$ and $h'(0) = 1$. In the particular case where Σ is a Kronecker product and A and B are chosen appropriately, the test statistic is

$$\xi_{CR} = T \sum_{i=1}^{s} h(\widehat{\lambda}_i)\,.$$

Choosing $h(\nu) = \nu$ results in what Robin and Smith (2000) referred to as the Wald variant of the characteristic root test statistic

$$\xi_{CR} = T \sum_{i=1}^{s} \widehat{\lambda}_i\,. \tag{23}$$

If $h(\lambda) = \log(1 + \lambda)$, the canonical correlations statistic of Anderson (1951) is recovered, as the eigenvalues of $\widehat{A}\widehat{\Phi}'\widehat{B}\widehat{\Phi}$, $\widehat{\lambda}$, and the eigenvalues of the matrix Ψ in expression (15), $\widehat{\nu}$, are related by $\widehat{\nu} = \widehat{\lambda}/(1 + \widehat{\lambda})$ (see the Appendix for further details).

4.3 Equivalence of Tests

In the particular case hen the asymptotic covariance matrix of $\mathrm{vec}\,\Phi$ is of the form $\Sigma = \Omega \otimes Q^{-1}$, the tests in the preceding subsection, namely the Singular Value Decomposition test of Kleibergen and Paap (2006), the Minimum Discrepancy test of Cragg and Donald (1993, 1997) and the Wald variant of the Characteristic Root test of Robin and Smith (2000) are all equal to the Wald test ξ_W in (20).

First note that choosing $\widehat{A} = \hat{\Omega}^{-1}$ and $\widehat{B} = \hat{Q}$ means that the Wald variant of the Characteristic Root test is simply T times the sum of the smallest s eigenvalues of $\widehat{\Theta}'\widehat{\Theta}$. Kleibergen and Paap (2006) advocate scaling $\widehat{\Phi}$ to $\widehat{\Theta}$ such that the estimated asymptotic covariance matrix of $\mathrm{vec}\,\widehat{\Theta}$ is the identity matrix. This can be done when estimating the parameters of a VECM by choosing $M = \hat{Q}^{1/2}$ and $N = \hat{\Omega}^{-1/2}$. The denominator of the SVD test statistic in expression (21) therefore reduces to an identity matrix. As a result, ξ_{SVD} is T times the sum of the squared s smallest singular values of $\widehat{\Theta}$.

Result 1: When $\Sigma = \Omega \otimes Q^{-1}$ and the scaling matrices are chosen appropriately, the Singular Value Decomposition test statistic in expression (21) is equal to the Characteristic Root test statistic in expression (23).

Proof: See Proposition 1 in Kleibergen and Paap (2006).

When Φ represents the parameters of a VECM, the fact that Σ is a Kronecker product also means that the Minimum Discrepancy test statistic is available analytically, so the objective function $\mathrm{vec}(\widehat{\Phi} - \Upsilon)'\widehat{\Sigma}^{-1}\mathrm{vec}(\widehat{\Phi} - \Upsilon)$ does not have to be minimized numerically over the set of all reduced-rank matrices. When $\widehat{\Sigma} = \hat{\Omega} \otimes \hat{Q}^{-1}$, the objective function may be re-expressed as

$$\begin{aligned} \mathrm{vec}(\widehat{\Phi} - \Upsilon)'\widehat{\Sigma}^{-1}\mathrm{vec}(\widehat{\Phi} - \Upsilon) &= \mathrm{tr}\left[\hat{\Omega}^{-1}(\widehat{\Phi} - \Upsilon)'\hat{Q}(\widehat{\Phi} - \Upsilon)\right] \\ &= \mathrm{tr}\left[(\widehat{\Theta} - \widetilde{\Upsilon})'(\widehat{\Theta} - \widetilde{\Upsilon})\right] \end{aligned}$$

where $\widetilde{\Upsilon} = \hat{Q}^{1/2}\Upsilon\hat{\Omega}^{-1/2}$. It follows that the minimum value of the objective function over the set of all Υ of rank $n - s$ is just the sum of the s smallest eigenvalues of $\widehat{\Theta}$.

Result 2: When $\Sigma = \Omega \otimes Q^{-1}$, the Minimum Discrepancy test statistic in expression (22) is equal to the Characteristic Root test statistic in expression (23).

Proof: See Theorem 3 in Cragg and Donald (1993).

Therefore, in the case of testing the rank of the short-run dynamics parameter matrix of a VECM, the Singular Value Decomposition test , the Minimum Discrepancy test, and the Characteristic Root tests discussed in the preceding subsection are all equivalent. It remains to show that the Wald test ξ_W in (20) is also equal to these tests.

Result 3: When $\Sigma = \Omega \otimes Q^{-1}$, the Wald test statistic in expression (19) is equal to the Characteristic Root test statistic in expression (23).

Proof: See Appendix.

5 Test Performance with Experimental and Actual data

In this section simulation evidence on the finite sample behavior of the ξ_{LR} and ξ_W tests for reduced rank is provided and used to re-examine an existing empirical study. Existing Monte Carlo evidence (see for example, Hecq et al., 2006) reports the size and power of the ξ_{LR} test in small samples but tends to focus on the possible effects of incorrectly specifying the number of cointegrating vectors and/or the lag order of the VECM. As well as adding in an extra test ξ_W the experimental design, the simulation in this paper is based not on synthetic data but is instead calibrated with data on the common components in the real output of six Latin American countries. This data is described in Hecq et al. (2006).[5]

5.1 Experimental Data: Latin American GDP

This section contains an experiment based on a stylized VECM(1) calibrated to the data on the logarithm of real GDP of six Latin American economies, namely Brazil, Venezuela, Mexico, Peru, Columbia and Chile, used by Hecq (2004). The cointegration rank is set at $r = 3$ and three common transitory components, $s = 3$, are imposed. An unrestricted

[5]A further simulation exercise based on the real per capita income of four industrial regions in the US as reported by Vahid and Engle (1993) was performed. As the conclusions were the same as for Latin American GDP, only the one experiment is reported

VECM(1) was estimated and the estimated cofeature vectors were used to ensure that the rank of $\Phi = [\alpha \; A_1]$ was 3. The VECM system is summarized by the following numerical values[6]

$$
\begin{aligned}
\alpha &= \begin{bmatrix} -0.26 & -0.28 & -0.77 \\ -0.07 & -0.20 & -0.19 \\ -0.06 & -0.06 & -0.31 \\ 0.14 & 0.03 & -0.11 \\ -0.08 & 0.06 & -0.73 \\ -0.20 & -0.22 & 0.03 \end{bmatrix}, \\
\beta' &= \begin{bmatrix} 1.00 & 0.00 & 0.00 & -1.81 & 0.88 & 0.13 \\ 0.00 & 1.00 & 0.00 & -1.54 & 0.98 & 0.06 \\ 0.00 & 0.00 & 1.00 & -1.61 & 1.12 & -0.25 \end{bmatrix} \\
A_1 &= \begin{bmatrix} -0.17 & 0.01 & 0.44 & -0.21 & 0.02 & 0.17 \\ -0.03 & -0.26 & 0.35 & -0.04 & 0.01 & 0.15 \\ -0.02 & 0.03 & 0.12 & -0.05 & 0.02 & 0.01 \\ 0.19 & -0.20 & 0.10 & 0.15 & 0.08 & -0.09 \\ 0.00 & 0.38 & -0.04 & -0.07 & 0.07 & -0.15 \\ -0.23 & -0.07 & 0.21 & -0.18 & -0.08 & 0.25 \end{bmatrix}
\end{aligned}
$$

The disturbances ε_t have mean zero and covariance matrix

$$
\Omega = 10^{-4} \begin{bmatrix} 6.86 & 1.99 & 1.74 & 1.25 & 3.72 & 0.26 \\ 1.99 & 19.15 & 3.54 & 2.73 & 5.47 & -0.10 \\ 1.74 & 3.54 & 2.95 & 0.04 & 1.35 & 1.10 \\ 1.25 & 2.73 & 0.04 & 5.75 & 3.75 & 1.46 \\ 3.72 & 5.47 & 1.35 & 3.75 & 17.18 & 2.28 \\ 0.26 & -0.10 & 1.10 & 1.46 & 2.28 & 2.18 \end{bmatrix}.
$$

The co-feature vectors for the experiment are

$$
\tau = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \\ -1.16 & -1.17 & -0.42 \\ -0.96 & -0.14 & -0.39 \\ -1.71 & -1.10 & -0.45 \end{bmatrix},
$$

where the last element of the second cofeature vector was changed from the estimated value of -4.10 to -1.10 so that the reduced-rank version of A_1 produced a stable VECM for all simulated data sets. Four sample sizes were considered - namely $T = 50$, 100, 200 and

[6]There is also a set of intercepts $\begin{bmatrix} 0.00 & -.01 & .02 & .01 & -.00 & .04 \end{bmatrix}'$.

Table 1: Simulation Exercise – Size

		ξ_{LR}	ξ_W
	$p = 2$	0.1764	0.3036
$T = 50$	$p = 3$	0.3445	0.6543
	$p = 4$	0.6036	0.9266
	$p = 2$	0.1115	0.1609
$T = 100$	$p = 3$	0.1577	0.2931
	$p = 4$	0.2371	0.4863
	$p = 2$	0.0794	0.0986
$T = 200$	$p = 3$	0.0985	0.1441
	$p = 4$	0.1197	0.2140
	$p = 2$	0.0551	0.0587
$T = 1{,}000$	$p = 3$	0.0591	0.0657
	$p = 4$	0.0600	0.0714

Size of the tests for $s = 3$ common factors for the simulation exercise using asymptotic 5% critical values. Data is simulated using the VECM(1) calibrated on the Latin American dataset used by Hecq (2004).

1,000. Computations were carried out using Matlab. 10,000 replications were used in the experiments, and the first 5,000 observations in each replication were discarded to remove dependence on initial observations. The sizes of the tests are presented in Table 1 while the empirical and asymptotic 5% critical values of these tests for the null hypothesis of $s = 3$ are presented in Table 2.

Two broad conclusions emerge from this analysis. First, both tests perform best in terms of size when the correct number of parameters is fitted, with the performance deteriorating as the number of redundant parameters is increased, that is, the order of the estimated VAR is higher than the true order. Second, the empirical 5% critical values for the tests of the null hypothesis that $s = 3$ (Table 2), suggests that it would be very misleading to use the asymptotic critical values in situations where the sample size is smaller than 200. These kinds of sample sizes are exactly those often encountered in macroeconometric applications and the test results should therefore be treated with extreme caution if they are based on asymptotic distributions.

Table 3 provides results on the (size-adjusted) power of the tests. To do this it is necessary

Table 2: Empirical and asymptotic critical values

		ξ_{LR}	ξ_W	Asy. CV
$T = 50$	$p = 2$	35.4279	42.7729	28.8693
	$p = 3$	66.1634	89.0690	50.9985
	$p = 4$	101.2946	156.3381	72.1532
$T = 100$	$p = 2$	32.8225	35.7242	28.8693
	$p = 3$	58.5733	66.5806	50.9985
	$p = 4$	85.5934	102.0727	72.1532
$T = 200$	$p = 2$	30.8788	32.1340	28.8693
	$p = 3$	54.9161	58.2540	50.9985
	$p = 4$	78.4305	84.7532	72.1532
$T = 1{,}000$	$p = 2$	29.2114	29.4278	28.8693
	$p = 3$	52.0685	52.6179	50.9985
	$p = 4$	73.2090	74.2449	72.1532

Empirical 5% critical values for the null hypothesis of $s = 3$ are displayed for the tests for different sample size and VAR order. The asymptotic critical values are drawn from a χ^2 distribution with 18 ($p = 2$), 36 ($p = 3$), and 54 ($p = 4$) degrees of freedom.

to have a variant of the model with $s = 2$ common transitory components. Again the singular value decomposition of Φ was computed as $\Phi = USV'$. The singular values in S were 1.3021, 0.8131, 0.5008, 0.000, 0.000 and 0.0000. For the first set of results (presented in Table 3) a rank 4 version of Φ was constructed by replacing the first zero singular value of Φ, contained in S, by 0.5008, giving S_0. The α and A_1 matrices were then found by partitioning $\Phi_0 = US_0V'$, after which the same tests for $s = 3$ common transitory components were performed.

It is apparent from the results of Table 3 that while the size properties of the LR test appear superior to that of the Wald test, the power of the latter is superior to the former. Thus if one can accurately determine the critical values for the Wald test, there would be gains to using it.

Table 3: Simulation Exercise – Power

		ξ_{LR}	ξ_W
$T = 50$	$p = 2$	0.8096	0.8353
	$p = 3$	0.5844	0.6410
	$p = 4$	0.4234	0.4745
$T = 100$	$p = 2$	0.9990	0.9993
	$p = 3$	0.9888	0.9918
	$p = 4$	0.9538	0.9702
$T = 200$	$p = 2$	1.0000	1.0000
	$p = 3$	1.0000	1.0000
	$p = 4$	1.0000	1.0000
$T = 1{,}000$	$p = 2$	1.0000	1.0000
	$p = 3$	1.0000	1.0000
	$p = 4$	1.0000	1.0000

Size-adjusted power of the tests for $s = 3$ common factors for the simulation exercise using empirical 5% critical values. Data is simulated using a variant of the VECM(1) calibrated on the Latin American dataset used by Hecq (2004) with $s = 2$ common transitory components. The fourth-largest singular value of Φ is set at 0.5008.

5.2 An Empirical Study: Application to Latin American GDP

This section investigates the presence of short-run and long-run interactions between the output of six Latin American economies, namely Brazil, Venezuela, Mexico, Peru, Columbia and Chile. To facilitate comparison with the work of Hecq (2004), Argentina is excluded from the analysis. The annual output data are extracted from the Total Economy Database and span the 59-year period 1950–2008. The analysis was conducted on the logarithm of the real GDP series for the six countries. The logarithms of real GDP for the six Latin American countries and also the growth rates of real GDP are plotted in Figure 1. The plot indicates that the variables in log-levels are trending while the growth rates appear to be stationary.

In implementing tests for the cointegration rank, a VAR with maximum order of four was fitted. As this is annual data it is unlikely that the dynamics of the system would require a longer lag length. Moreover, with $n = 6$ variables, the number of coefficients to be estimated soon becomes prohibitive if a longer lag structure is used. This model also included a restricted deterministic trend in the long-run specification.

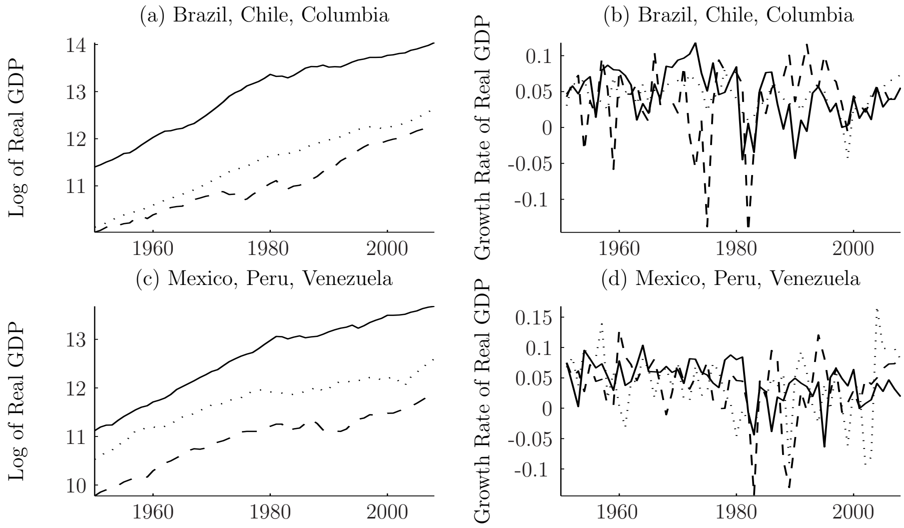

Figure 1: Logarithm of Latin American GDP and Growth Rates. Top row of the figure shows the results for Brazil (solid line), Chile (dashed line) and Columbia (dotted line). The second row of the figure shows the results for Mexico (solid line), Peru (dashed line) and Venezuela (dotted line).

Table 4: Tests for cointegrating rank

Test	$p = 2$	$p = 3$	$p = 4$
	Model 1		
Trace	$r = 1$	$r = 4$	$r = 5$
Max. Eigenvalue	$r = 0$	$r = 3$	$r = 2$
	Model 2		
Trace	$r = 1$	$r = 5$	$r = 6$
Max. Eigenvalue	$r = 1$	$r = 1$	$r = 3$

Summary of tests for the cointegrating rank (r) for Latin American GDP data for 1950 to 2008 based in two alternative models and various choices for the lag order of the test VAR. Model 1 has a constant in both the cointegration equation and the VAR. Model has a constant and trend in the cointegration equation and a constant in the VAR.

Experimentation with models containing two, three and four lags in the test VAR, combined with an assumption about the presence or absence of a deterministic trend in the cointegration space, revealed a fairly robust pattern. Discounting the results for the lag order of $p = 2$, which may be too short, the trace test statistic probably overestimated the number of cointegration vectors, while the maximal eigenvalue test suggested a cointegration rank of $r = 2$ or $r = 3$. It is very unlikely that all the GDP series for the seven Latin American countries are trend stationary so the possibility of $r = 6$ being chosen based on Table 4 is discounted. The conclusion drawn from the balance of the evidence is that the choice of $r = 3$ is probably the best choice. Selecting $r = 3$ rather than $r = 2$ is also motivated in part by the (limited) simulation evidence available which suggests that underestimating the cointegration rank can lead to misleading inference in terms of the common transitory components.

Hecq (2004) argues that there are three permanent and three transitory shocks within the six Latin American economies, in other words there are as many long-run co-movements as short-run common transitory components. The likelihood ratio and Wald tests are now applied to the Latin American data, based on the assumption of $r = 3$. Given that $r + s \leq n$ it follows that the maximum number of common transitory component to be tested for is $s = 3$. The results of these tests are summarized in Table 5. Once again, tests are presented for different lag orders in the VAR ($p = 2$, 3, 4) and for two different models, namely a model with a constant in both the cointegrating equation (and the VAR) and a model with both a constant and a trend in the cointegrating equation along with a constant in the VAR. The latter is the model employed by Hecq (2004) and appears to be favoured marginally by the log-likelihood values returned in the estimation.

In line with the findings of Hecq (2004), there is evidence to support the hypothesis of three common transitory components, that is $s = 3$. The LR and Wald statistics for the VARs of orders $p = 2$ and $p = 3$ all seem to point to the conclusion that there are $s = 3$ common factors in the data. In the simulation exercise reported in Table 2, both these tests were shown to have particularly large critical values in small samples when applied to a fourth-order VAR, namely 101.29 for the LR test and 156.34 for the Wald test. In the case $p = 4$, while reference of the tests to the asymptotic critical values would suggest that the null of

Table 5: Latin American GDP

H_0	5% Critical Value	ξ_{LR}	ξ_W
Model 1: Constant in Cointegrating Equation, Constant in Test VAR			
$p = 2$			
$s = 1$	9.4877	1.5126	1.5329
$s = 2$	18.3070	12.3002	13.4088
$s = 3$	28.8693	25.3843	28.1164
$p = 3$			
$s = 1$	18.3070	2.4143	2.4671
$s = 2$	33.9244	14.2891	15.6948
$s = 3$	50.9985	31.9363	36.4393
$p = 4$			
$s = 1$	26.2962	9.9603	10.9192
$s = 2$	48.6024	35.5980	43.5798
$s = 3$	72.1532	81.8863	116.1848
Model 2: Constant and Trend in Cointegrating Equation, Constant in Test VAR			
$p = 2$			
$s = 1$	9.4877	1.3552	1.3715
$s = 2$	18.3070	8.2545	8.7056
$s = 3$	28.8693	19.2158	20.7918
$p = 3$			
$s = 1$	18.3070	1.6921	1.7180
$s = 2$	33.9244	11.1126	11.9772
$s = 3$	50.9985	28.8376	32.8282
$p = 4$			
$s = 1$	26.2962	9.3572	10.2003
$s = 2$	48.6024	40.2824	51.7067
$s = 3$	72.1532	82.9843	116.2563

Tests for common transitory components in Latin American GDP data for 1950 to 2008 based on the assumption of $r = 3$ cointegrating vectors.

$s = 3$ is rejected in favour of $s = 2$, this is not so if one uses the empirical critical values for $T = 50$. This is an interesting result, as it adds to the cautionary tale outlined previously that the use of an asymptotic critical values in small-sample macroeconomic examples can lead to incorrect inference.

6 Common Transitory Components in DSGE Models

Much quantitative work in macroeconomics is now conducted in terms of the estimation of DSGE models rather than via statistical models such as a VECM. An interesting question therefore is whether common transitory components are present if such a model is used to represent a macroeconomy. To answer that question it is necessary to map a DSGE model into a VECM so as to apply the tests developed earlier. To date a mapping of this sort does not appear in literature.

The focus here will be upon DSGE models that have a single permanent component driving them. Generally this will be the logarithm of the level of technology $a_t = \ln A_t$. There are models now that have two or more permanent components but these are mainly to capture changes in relative prices. The methodology that is presented here does have a simple extension to those cases, as should be evident.

DSGE models have a structure that involves a set of equations summarizing inter-temporal decisions - the Euler equations - and some other equations, such as the national income identity, that could be either static or dynamic. An example of the first would be the consumption Euler equation

$$C_t = \beta E_t C_{t+1} R_{t+1} ,$$

where C_t is the level of consumption and R_t is a real interest rate. When variables are stationary the equation can be re-expressed in terms of ratios of the variables to their steady state positions C^* and R^*, that is

$$\frac{C_t}{C^*} = \beta R^* E_t \frac{C_{t+1}}{C^*} \frac{R_{t+1}}{R^*} ,$$

but, when variables are non-stationary, some other divisor has to be used. Traditionally in DSGE models this has been the level of technology, so that the consumption Euler equation becomes

$$\frac{C_t}{A_t} = \delta R^* E_t \frac{C_{t+1}}{A_{t+1}} \frac{A_{t+1}}{A_t} \frac{R_{t+1}}{R^*} \,.$$

After log-linearization, the equation is

$$c_t - a_t = E_t[c_{t+1} - a_{t+1} + \Delta a_{t+1}] + E_t r_{t+1} - r^*,$$

where the lower case letters represent the logs of the upper case ones. Generally technology growth is assumed to be an exogenous AR(1) process

$$\Delta a_t = \rho_a \Delta a_{t-1} + \varepsilon_{at},$$

so that $E_t \Delta a_{t+1} = \rho_a \Delta a_t$, making the linearized consumption Euler equation:

$$c_t - a_t = E_t[c_{t+1} - a_{t+1}] + \rho_a \Delta a_t + E_t r_{t+1} - r^* \,.$$

Other equations can be treated in the same way. If therefore the variables ψ_t are defined as $c_t - a_t$, $r_t - r^*$ and so on, it is possible to represent a DSGE model in terms of the following set of structural equations

$$B_0 \psi_t = B_1 \psi_{t-1} + C_1 E_t \psi_{t+1} + G_1 e_t + G_2 \Delta a_t \,,$$

where the model (non-technology) shocks, e_t, are assumed to follow a first order VAR process:

$$e_t = \Phi_e e_{t-1} + \varepsilon_{et} \,. \tag{24}$$

Now, the division of $I(1)$ variables by A_t is often referred to as "stationizing" the variables, and it is clear that variables appearing in ψ_t, such as $c_t - a_t$, will be co-integrating errors.

Let $z_t = [y_t,\, a_t]'$ be the n $I(1)$ variables of the DSGE model, so that $\psi_t = \beta' z_t$ are EC terms when the columns of β are the r cointegrating vectors. Because of the "stationizing" transformation in DSGE models there are generally $r = n - 1$ cointegrating vectors of the form

$$\beta' = \begin{bmatrix} 1 & 0 & . & . & -1 \\ 0 & 1 & . & . & -1 \\ 0 & 0 & . & . & . \\ & & & 1 & -1 \end{bmatrix} .$$

For convenience, it is assumed initially that all the y_t are $I(1)$ leaving the issue of adapting the methodology to allow some of the y_t to be $I(0)$ to a later stage.

The "stationized" DSGE model is solved to give

$$\psi_t = D_1\psi_{t-1} + D_a\Delta a_t + D_e e_t. \tag{25}$$

Using the processes assumed for the shocks in equation (25) the system becomes

$$\psi_t = D_1\psi_{t-1} + D_a\rho_a\Delta a_{t-1} + D_a\varepsilon_{at} + D_e\Phi_e e_{t-1} + D_e\varepsilon_{et}. \tag{26}$$

Often the output from packages such as DYNARE appear in this way under the nomenclature of "policy and transition functions", enabling one to use that output to recover the implied parameters of (25), since ρ_a and Φ_a are given.

At this point it is necessary to impose the restriction that there are enough shocks in the system, so it is assumed that D_e has full column rank, and then the Moore-Penrose generalized inverse of D_e, given by $D_e^+ = (D_e'D_e)^{-1}D_e'$, exists. Given this assumption the shocks can be recovered from (25) as

$$e_t = D_e^+(\psi_t - D_1\psi_{t-1} - D_a\Delta a_t). \tag{27}$$

Replacing e_{t-1} in (26) with its value from (27) gives

$$\psi_t = H_1\psi_{t-1} + H_2\psi_{t-2} + H_3\Delta a_{t-1} + D_e\varepsilon_{et} + D_a\varepsilon_{at}\,,$$

where

$$\begin{aligned}
H_1 &= D_1 + D_e\Phi_e D_e^+, \\
H_2 &= -D_e\Phi_e D_e^+ D_1, \\
H_3 &= D_a\rho_a - D_e\Phi_e D_e^+ D_a\,.
\end{aligned}$$

In turn this equation can be written as

$$\begin{aligned}
\psi_t &= (H_1 + H_2)\psi_{t-1} - H_2\Delta\psi_{t-1} + H_3\Delta a_{t-1} + D_e\varepsilon_{et} + D_a\varepsilon_{at} \\
&= (H_1 + H_2)\psi_{t-1} - (H_2\beta' - H_3 S_a)\Delta z_{t-1} + D_e\varepsilon_{et} + D_a\varepsilon_{at},
\end{aligned} \tag{28}$$

where S_a selects a_t from z_t i.e. $a_t = S_a z_t$.

A VECM in z_t may be written as

$$\begin{aligned}
\Delta z_t &= \alpha\beta' z_{t-1} + A_1\Delta z_{t-1} + v_t \\
\Longrightarrow \quad \psi_t &= (I + \beta'\alpha)\psi_{t-1} + \beta' A_1\Delta z_{t-1} + \beta' v_t\,.
\end{aligned} \tag{29}$$

Comparing (29) and (28) we see that

$$\beta'\alpha = (H_1 + H_2) - I \tag{30}$$

$$\beta' A_1 = -(H_2\beta' - H_3 S_a)\,. \tag{31}$$

To recover α and A_1 from these relations we need to recognize that Δa_t is a strongly exogenous process. This means that the elements in α corresponding to Δa_t are zero. Since there are r of these, this leaves $(n-1)\times r$ unknowns in α. In the standard DSGE set-up $r = n-1$ so the $r^2 = (n-1)^2$ unknowns can be determined from the r^2 equations in (30). Strong exogeneity of Δa_t also means that A_1 has zero elements in it and one of the elements is the known ρ_a. This leaves $(n-1)\times n$ unknowns to be determined by the $r \times n$ linear equations in (31). Again, in the standard case $r = n-1$, and so there are enough equations to determine A_1.

Now, in the event that there are both $I(0)$ and $I(1)$ variables in y_t these results need to be generalized. This is relatively easily accomplished by treating the m $I(0)$ variables in y_t as if they were $I(1)$ but with m extra co-integrating vectors that have unity in the column corresponding to the $I(0)$ variable and zeroes elsewhere. Thus, if the second variable in y_t is $I(0)$, we would add on a co-integrating vector of the form $\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \end{pmatrix}$.

To illustrate the working of this procedure, it is applied to a variant of the small open-economy model set out in Lubik and Schorfheide (2007) as implemented by Hodge et al. (2008):

$$\tilde{\xi}_t = E_t\tilde{\xi}_{t+1} - \chi(R_t - E_t\pi_{t+1}) + \chi\rho_a\Delta a_t + \alpha\chi E_t\Delta\tilde{q}_{t+1} + (\frac{\chi}{\tau} - 1)E_t\Delta\tilde{y}^*_{t+1} \tag{32}$$

$$\tilde{\pi}_t = \beta E_t(\tilde{\pi}_{t+1}) + \alpha\beta E_t\Delta\tilde{q}_{t+1} - \alpha\Delta\tilde{q}_t + \frac{\kappa}{\chi}(\tilde{\xi}_t - (1 - \frac{\chi}{\tau})\tilde{y}^*_t) \tag{33}$$

$$E_t\Delta\tilde{e}_{t+1} = \tilde{\pi}_t - (1-\alpha)\Delta\tilde{q}_t - \tilde{\pi}^*_t \tag{34}$$

$$\tilde{R}_t = \rho_R\tilde{R}_{t-1} + (1-\rho_R)(\psi_1\tilde{\pi}_t + \psi_2\tilde{\xi}_t) + \varepsilon_{R,t} \tag{35}$$

$$\Delta\tilde{q}_t = \rho_{\Delta q}\Delta\tilde{q}_{t-1} + \varepsilon_{\Delta q,t} \tag{36}$$

$$\tilde{y}^*_t = \rho_{y^*}\tilde{y}^*_{t-1} + \varepsilon_{y^*_t,t} \tag{37}$$

$$\tilde{\pi}^*_t = \rho_{\pi^*}\tilde{\pi}^*_{t-1} + \varepsilon_{\pi^*_t,t} \tag{38}$$

$$\Delta a_t = \rho_a\Delta a_{t-1} + \varepsilon_{a,t} \tag{39}$$

where $\tilde{\xi}_t = \xi_t - a_t$ and other variables are taken to be log deviations from constant steady state values.[7] Here ξ_t is the log of output and will be $I(1)$, e_t is the log of the exchange rate, R_t is the (nominal) rate of interest, π_t is the rate of inflation, q_t is the (observed) log of the terms of trade, y_t^* is the log of foreign output, π_t^* is the log of foreign inflation, and a_t is the log of the level of technology. The parameter χ is given by $\tau + \alpha(2-\alpha)(1-\tau)$, where α is the share of imported goods in consumption, and $\tau = 1/\sigma$ comes from a CARA utility function of the form $(C_t/A_t)^{1-\sigma}/(1-\sigma) - N_t$.

In terms of equation (25), $\psi_t = \{\tilde{\xi}_t, \tilde{\pi}_t, \Delta\tilde{e}_t, \tilde{R}_t, \Delta\tilde{q}_t\}$ and the non-technology shocks are $e_t = \{\varepsilon_{R,t}\ \varepsilon_{\Delta q,t} \tilde{y}_t^*, \tilde{\pi}_t^*\ \}$. Values of the parameters taken from Hodge et al. (2008) are

$$\begin{aligned}
\tau &= .5, \alpha = .2, \rho_a = .29, \kappa = .42, \rho_R = .81, \psi_1 = 1.62, \\
\psi_2 &= .4, \rho_{\Delta q} = .57, \rho_{\pi^*} = .53, \rho_{y^*} = .92.
\end{aligned}$$

With the parameter values given above the solution of this model is

$$\begin{aligned}
\tilde{\xi}_t &= -.99\tilde{R}_{t-1} + .05\Delta a_{t-1} + .08\Delta q_{t-1} - .20\tilde{y}^*_{t-1} + .14\varepsilon_{\Delta q,t} - 1.2\varepsilon_{R,t} \\
&\quad -.22\varepsilon_{y^*_{,t}} + .17\varepsilon_{a,t} \\
\tilde{\pi}_t &= -.1.04\tilde{R}_{t-1} + .03\Delta a_{t-1} + .01\Delta q_{t-1} - .24\tilde{y}^*_{t-1} - .17\varepsilon_{\Delta q,t} - 1.28\varepsilon_{R,t} \\
&\quad -.26\varepsilon_{y^*_{,t}} + .09\varepsilon_{a,t} \\
R_t &= .42\tilde{R}_{t-1} + .01\Delta a_{t-1} + .00\Delta q_{t-1} + .06\tilde{y}^*_{t-1} + .01\varepsilon_{\Delta q,t} + .51\varepsilon_{R,t} \\
&\quad +.06\varepsilon_{y^*_{,t}} + .04\varepsilon_{a,t} \\
\Delta e_t &= -.53\tilde{\pi}^*_{t-1} - .1.04\tilde{R}_{t-1} + .03\Delta a_{t-1} - .47\Delta q_{t-1} + .24\tilde{y}^*_{t-1} - .82\varepsilon_{\Delta q,t} \\
&\quad -\varepsilon_{\pi^*_{,,t}} - 1.28\varepsilon_{R,t} + .26\varepsilon_{y^*_{,t}} + .09\varepsilon_{a,t} \\
\Delta\tilde{q}_t &= \rho_{\Delta q}\Delta\tilde{q}_{t-1} + \varepsilon_{\Delta q,t}
\end{aligned}$$

This expression has the form of equation (26), thus enabling the matrices D_a, D_e and D_1 to be recovered. Hence the VECM representation of the DSGE model can be constructed. In other words, the parameter matrices α and A_1 implied by the DSGE model can be found, thereby allowing the rank tests for common transitory components given earlier to

[7] The terms $\chi\rho_a z_t$ and $\alpha\chi E_t\Delta\tilde{q}_{t+1}$ are not the same as in Lubik and Schorfheide (2007) but the corrected version in Hodge et al. (2008).

be applied. In this particular case, the matrix has rank six, as needed if there are to be no common transitory dynamics among the five observable variables $\{\xi_t, \pi_t, \Delta e_t, R_t, \Delta q_t\}$ plus the technology variable a_t. If, however, technology growth is restricted to have no serial correlation, $\rho_a = 0$, and the matrix becomes of rank five.

7 Conclusion

This paper has argued that tests for common transitory factors in a model that has a VECM representation involves testing if the rank of the matrix containing the short-run dynamics coefficients is deficient. It was also demonstrated that, in this particular context, tests for reduced rank of the coefficient matrix are identical to a Wald variant of the commonly used likelihood ratio approach to testing for common transitory components. It was also shown that the Wald test appears to have more power than the popular LR test that is the workhorse of the current literature. Finally, it was demonstrated how a DSGE model with permanent shocks can be converted into a VECM so that the rank tests may be applied.

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Appendix: Proof of Result 3

First note that as the eigenvalues of

$$\Psi = (W_1'W_1)^{-1}W_1'W_2(W_2'W_2)^{-1}W_2'W_1 = (\widehat{\Phi}'\hat{Q}\widehat{\Phi} + \hat{\Omega})^{-1}\widehat{\Phi}'(\hat{Q})\widehat{\Phi}$$

from expression (15) are a.s. distinct and positive. Therefore $\widehat{\delta}$, representing the eigenvectors corresponding to the s smallest eigenvalues of Ψ, a.s. has full column rank s. Therefore, any quadratic form $\widehat{\delta}'A\widehat{\delta}$ is a.s. positive-definite and symmetric provided A is positive-definite and symmetric.

The Wald test statistic may be re-expressed as

$$\begin{aligned} \xi_W &= T\,\mathrm{vec}(\widehat{\Phi}\widehat{\delta})'\left[(\widehat{\delta}'\otimes I_k)\widehat{\Sigma}(\widehat{\delta}\otimes I_k)\right]^{-1}\mathrm{vec}(\widehat{\Phi}\widehat{\delta}) \\ &= T\,\mathrm{tr}\left[(\widehat{\delta}'\hat{\Omega}\widehat{\delta})^{-1}\widehat{\delta}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\delta}\right] \end{aligned}$$

using the fact that $\widehat{\Sigma} = \hat{\Omega}\otimes(\hat{Q})^{-1}$.

To show that the Wald test statistic is identical to the SVD, CR and MD test statistics, it remains to establish that the eigenvalues of the bracketed matrix are the s smallest eigenvalues of $\hat{\Omega}^{-1}\widehat{\Phi}'\hat{Q}\widehat{\Phi}$, as $\hat{\Omega}^{-1}\widehat{\Phi}'\hat{Q}\widehat{\Phi}$ and $\widehat{\Theta}'\widehat{\Theta}$ have the same eigenvalues.

Let $B = \widehat{\Phi}'\hat{Q}\widehat{\Phi}$ and $A = \hat{\Omega}^{-1}$. The eigendecomposition of $\hat{\Omega}^{-1}\widehat{\Phi}'\hat{Q}\widehat{\Phi}$ can be expressed as

$$\Xi'AB\Xi = \Lambda\,.$$

Noting that $\Psi = (B + A^{-1})^{-1}B$, it follows that

$$\begin{aligned} \Xi'(B + A^{-1})^{-1}B\Xi &= \Xi'(A^{-1}\Xi\Lambda\Xi' + A^{-1})^{-1}A^{-1}\Xi\Lambda \\ &= \Xi'[A^{-1}\Xi(\Lambda + I)\Xi']^{-1}A^{-1}\Xi\Lambda \\ &= (\Lambda + I)^{-1}\Lambda\,. \end{aligned}$$

Therefore, the canonical correlations $\widehat{\nu}$ and eigenvalues of $\widehat{\Theta}'\widehat{\Theta}$ are related by $\widehat{\nu} = \widehat{\lambda}/(1+\widehat{\lambda})$. It follows that each of the eigenvectors corresponding to the s smallest eigenvalues of Ψ, forming the columns of $\widehat{\delta}$, are just scalar multiples of the eigenvectors corresponding to the s smallest eigenvalues of $\widehat{F}\widehat{F}\widehat{\Phi}'\hat{Q}\widehat{\Phi}$, forming the columns of $\widehat{\tau}$. To account for arbitrary normalizations of $\widehat{\delta}$, such as choosing to represent $\widehat{\delta}'$ in reduced row echelon form, it suffices to set $\widehat{\delta} = M\widehat{\tau}$ for some non-singular M.

This means that the Wald statistic may be re-expressed as

$$\begin{aligned}\xi_W &= T\,\mathrm{tr}\left[(\widehat{\xi}'M'\hat{\Omega}M\widehat{\xi})^{-1}\widehat{\xi}'M'\widehat{\Phi}'\hat{Q}\widehat{\Phi}M\widehat{\xi}\right] \\ &= T\,\mathrm{tr}\left[(\widehat{\xi}'\hat{\Omega}\widehat{\xi})^{-1}\widehat{\xi}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\xi}\right]\end{aligned}$$

by the self-similarity of the ratio of these two quadratic forms with respect to M.

From the eigendecomposition of $\hat{\Omega}^{-1}\widehat{\Phi}'\hat{Q}\widehat{\Phi}$ it can also be shown that

$$\Xi'\hat{\Omega}\Xi = \Xi'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\Xi\Lambda^{-1}\,.$$

This means that $(\widehat{\tau}'\hat{\Omega}\widehat{\tau})^{-1} = (\widehat{\tau}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\tau})^{-1}\Lambda_s$ where $\Lambda_s = \mathrm{diag}\{\widehat{\lambda}_1, \ldots, \widehat{\lambda}_s\}$.

Therefore

$$\begin{aligned}\xi_W &= T\,\mathrm{tr}\left[(\widehat{\tau}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\tau})^{-1}\Lambda_s(\widehat{\tau}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\xi})\right] \\ &= T\,\mathrm{tr}\left[\Lambda_s(\widehat{\xi}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\tau})(\widehat{\tau}'\widehat{\Phi}'\hat{Q}\widehat{\Phi}\widehat{\tau})^{-1}\right] \\ &= T\textstyle\sum_{i=1}^{s}\widehat{\lambda}_i\end{aligned}$$

as required.

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